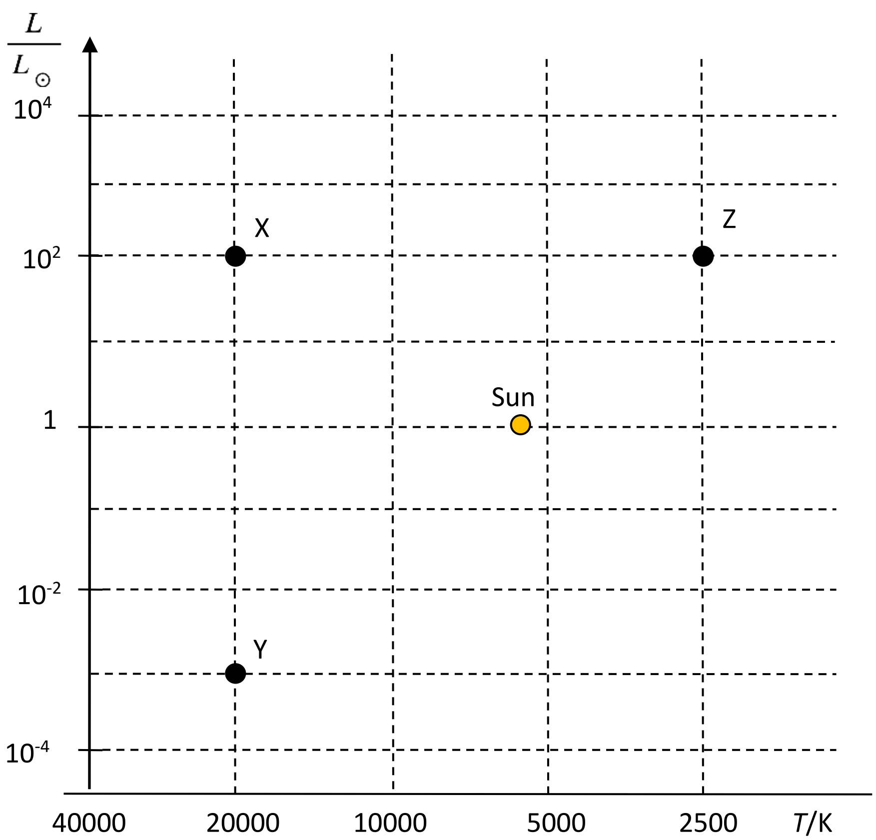
Teacher notes

Topic E

The HR diagram



(a) Identify (i) the main sequence, (ii) the region of red giants, (iii) the region of white dwarfs and (iv) the instability region.

(b) X and Y have the same luminosity even though X has a much larger temperature. Explain this observation.

(c) What is the ratio of radii $R_Z : R_X : R_Y$ for Z and X and Y?

(d) Describe and draw the evolutionary path of the Sun and of star X.

(e) Describe how star X and star Y maintain equilibrium.

Answers

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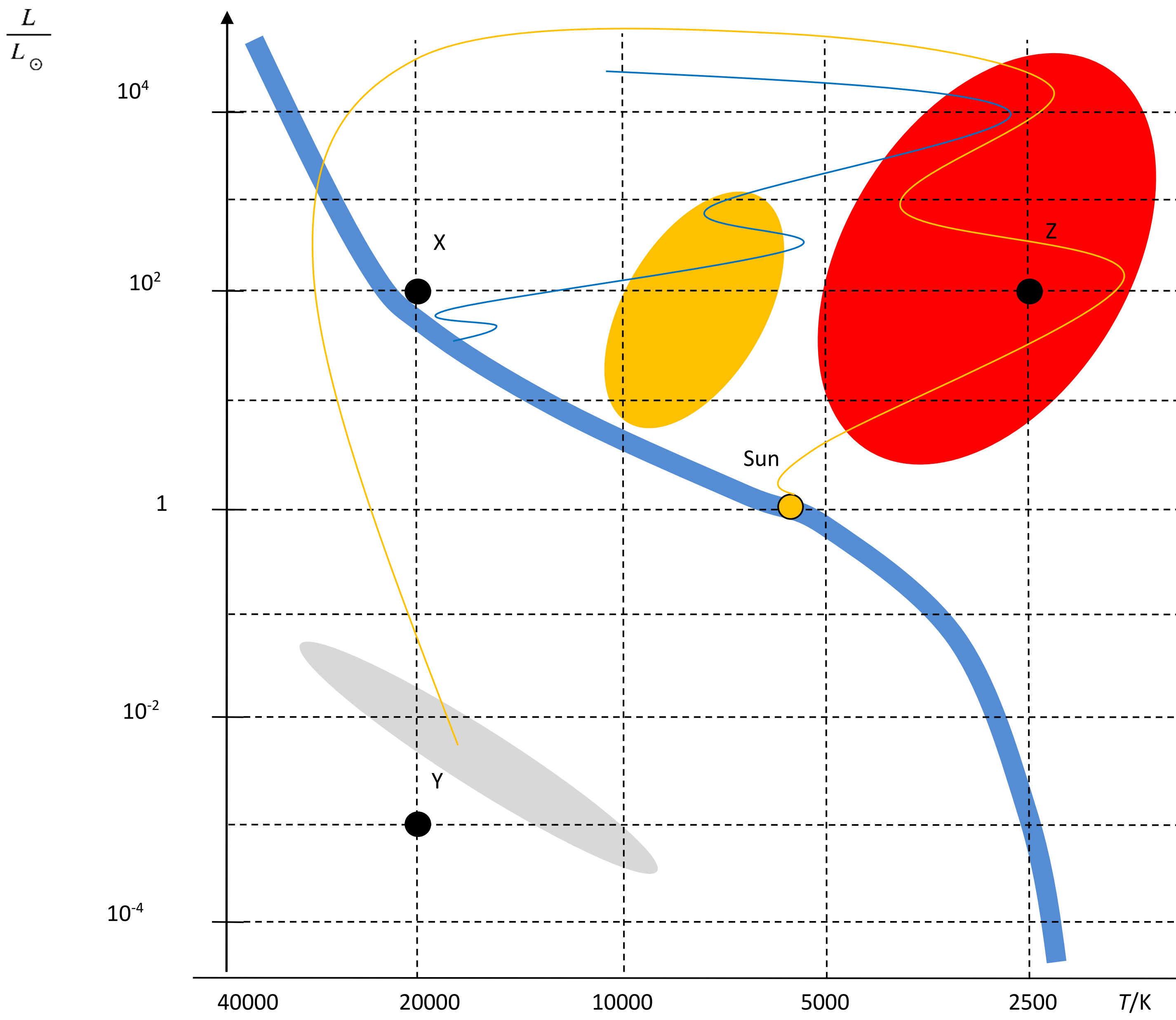
(a)



$\frac{L}{L_\odot}$
10^4
10^2
1
10^{-2}
10^{-4}
X
Z
Sun
Y
40000
20000
10000
5000
2500
T/K

(b) Z must have a much larger surface area.

(c) $\frac{\sigma 4\pi R_Z^2 \times 2500^4}{\sigma 4\pi R_X^2 \times 20000^4} = 1 \Rightarrow \frac{R_Z}{R_X} = (\frac{20000}{2500})^2 = 64$; $\frac{\sigma 4\pi R_X^2 \times 20000^4}{\sigma 4\pi R_Y^2 \times 20000^4} = 10^5 \Rightarrow \frac{R_X}{R_Y} = \sqrt{10^5} \approx 320$. Hence

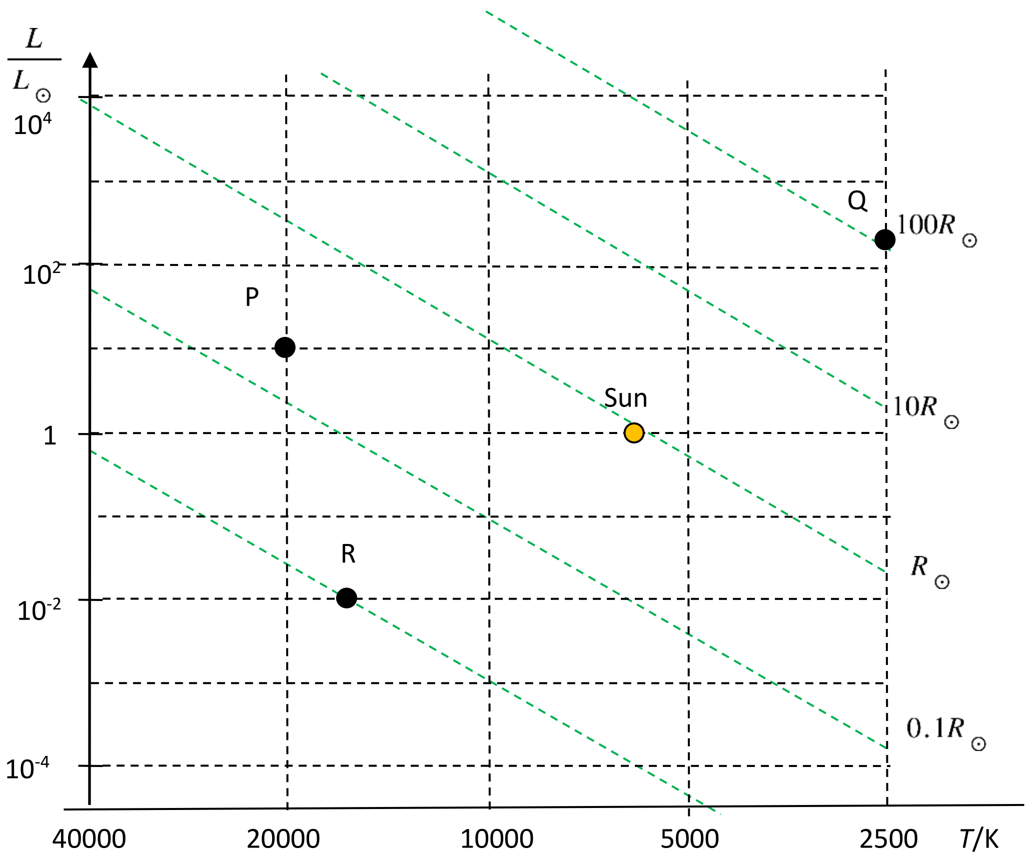
$R_Z : R_X : R_Y \approx 20500 : 320 : 1$

(d) See diagram.

(e) X: pressure (gas pressure and radiation pressure) created by the energy produced in nuclear fusion

Y: electron degeneracy pressure.

Why are lines of constant stellar radius straight lines on the HR diagram?



Since $L = \sigma 4\pi R^2 T^4$ it follows that $\log\frac{L}{L_\odot} = \log\left(\frac{R^2}{R_\odot^2}\frac{T^4}{T_\odot^4}\right) = 2\log\left(\frac{R}{R_\odot}\right) + 4\log\left(\frac{T}{T_\odot}\right)$. We take logs because the HR diagram is a plot of log *L* versus log *T*. For *R* = constant,

$$\log \frac{L}{L_{\odot}} = c + 4\log\left(\frac{T}{T_{\odot}}\right)$$

This would be a straight line with positive slope on the HR diagram. But *T* is increasing to the left so this makes the straight line have a negative gradient. The gradient is – 4.

For the line through the Sun *c* = 0.

We can now ask for: the luminosity of Q, the temperature of R and the radius of P. (This is NOT something that could be asked on an IB exam but it could be useful to someone doing an IA or EE.)

For Q:

$\log \frac{L}{L_{\odot}} = 2\log(100) + 4\log\left(\frac{2500}{5780}\right) = 2.544$. Hence $L = 10^{2.544} = 3.5 \times 10^{2} L_{\odot}$.

For R:

$\log 10^{-2} = 2\log(10^{-2}) + 4\log\left(\frac{T}{5780}\right) \Rightarrow \log\left(\frac{T}{5780}\right) = \frac{1}{2}$. Hence $\frac{T}{5780} = 10^{\frac{1}{2}} \Rightarrow T = 1.8 \times 10^{4}\ \text{K}$.

For P:

$\log 10 = 2\log\left(\frac{R}{R_{\odot}}\right) + 4\log\left(\frac{20000}{5780}\right) \Rightarrow \log\left(\frac{R}{R_{\odot}}\right) = -0.578$. Hence $\frac{R}{R_{\odot}} = 10^{-0.587} \Rightarrow R = 0.26 R_{\odot}$.

These results are consistent with the diagram.